Wing Geometry Effects on Leading Edge Vortices

R. M. Kulfan,
Boeing Commercial Airplane Co.,
Seattle, Wash.

AIAA AIRCRAFT SYSTEMS AND TECHNOLOGY MEETING
August 20-22, 1979 / New York, New York
Abstract
Leading-edge vortices that form at off-design conditions can profoundly influence the aerodynamic characteristics of highly swept slender wings. A straightforward method that can predict wing geometry effects on the progressive spanwise development of leading-edge vortices is presented. This method, which can account for sweep, camber, twist, airfoil shape, flap deflections, and planform shape, is shown to be a useful tool for aerodynamic design studies. It is shown that suppressing the leading-edge vortex, particularly over the inboard portion of the wing, can cause considerable reduction in drag due to lift. Results of wing geometry parametric studies are shown.

1.0 Introduction

Aircraft capable of extended range while cruising at supersonic Mach numbers typically have thin, highly swept-slicker wings. These wings are of interest because they have the potential, according to theory, to obtain relatively low drag at supersonic lifting conditions.

Familiar characteristics of these wings are illustrated in figure 1.Low drag is achieved at the design condition with attached flow. At off-design conditions, the flow is dramatically changed by the sudden formation of a pair of rather stable leading-edge vortices. If the wing is very sharp and thin, the vortices spring from the entire leading edge. If the wing airfoil sections have rounded noses, the vortices first appear near the wing tip and then move progressively inboard with increasing angle of attack. These leading-edge vortices have a profound influence on the wing pressure distribution and, hence, on the aerodynamic performance, stability characteristics, and structural design loads.

Numerous experimental and theoretical studies have provided a fundamental understanding of the nature of these vortices on rather simple thin-wing geometries. In practice, supersonic wing designs, as shown in figure 2, are becoming increasingly more sophisticated through the use of strakes, curved leading edges, wing airfoil shapes that vary across the span, drooped leading edges, and wing camber and twist, as well as variable cruise flap deflections. All of these have an effect on the development and growth of the leading-edge vortices. It is important to understand the effects of wing geometry and flight conditions on the formation and control of these leading-edge vortices in order to develop efficient configurations, as well as to assess their aerodynamic characteristics.

Recently, a straightforward method to predict the effects of airfoil shape on the progressive growth of leading-edge vortices was developed. In this paper, that method is extended to account for the effects of wing camber, twist, and flap deflections. The use of the method for conducting parametric design studies is discussed. Additionally, it is shown that, with the exception of very highly swept slender wings, minimum drag due to lift is achieved with attached flow. Low drag due to lift, however, can still be obtained with vortex lift, provided that the leading-edge vortex is restricted to the outer portions of the wing. Hence, aerodynamic designs that restrict the inboard movement of the leading-edge vortex should result in low drag due to lift. Results of using this method to investigate effects of various individual wing geometry parameters on the development of the leading-edge vortex are summarized.

In section 2.0, the basic characteristics of leading-edge vortices on sharp thin slender wings are discussed. The leading-edge suction analogy that forms the basis for the method presented in this paper is discussed in section 3.0. The simple and straightforward technique to predict the effect of pointed nose and round nose airfoils on leading-edge vortex development, and on the associated forces, is summarized in section 4.0. In section 5.0, this method is extended to account for wing warping (e.g., camber, twist, flap deflections) effects. In section 6.0, pressure data obtained on highly swept, flat and twisted wings are used to illustrate the ability of the method to predict the effects of wing twist and leading-edge flap deflections on the vortex development. In section 7.0, the prediction of drag due to lift with vortex flow is discussed. In section 8.0, the drag due to lift obtained with attached flow is compared to the drag due to lift obtained with vortex flow. It is concluded in this section that, except for very slender wings, the lowest drag due to lift is achieved with attached flow.

In the remaining sections, results of parametric studies using the prediction method are summarized. The objective is to provide an understanding of the influence of various wing geometry parameters on leading-edge vortices. The powerful effect of airfoil nose radius is discussed in section 9.0. Planform sweep, taper ratio, and notch ratio effects are summarized in section 10.0. Spanwise thickness variations are considered in section 11.0. Wing warp effects including twist, flap deflections, and camber are discussed in section 12.0.

2.0 Characteristics of Leading-Edge Vortices on Thin, Sharp-Edge Wings

Copyright: American Institute of Aeronautics and Astronautics, Inc., 1979. All rights reserved.
A separation vortex is well understood. When a highly swept wing is at an angle of attack, a dividing streamline is formed on the lower surface of the wing. This dividing streamline is similar to the forward stagnation point in two-dimensional flow. The flow inboard of the dividing streamline travels aft, and is swept past the wing trailing edge by the streamwise component of velocity. Lower surface flow forward of the dividing streamline travels from the lower surface around the leading edge to the upper surface.

The expansion of the flow going around the leading edge results in a very high negative pressure and a subsequent steep adverse pressure gradient. The steep adverse pressure gradient can readily cause the three-dimensional boundary layer to separate from the surface. When separation occurs, the lower surface boundary layer leaves the wing along the leading edge and rolls up into a region of concentrated vorticity, which is swept back over the upper surface of the wing. The strong vorticity, however, draws air above the wing into the spiral sheets and, thereby, induces a strong sidewash on the upper surface of the wing that is directed toward the leading edge. This leads to a minimum pressure under the leading-edge vortices on the upper surface of the wing. An increase in lift at a given angle of attack results, and it is this increase that is usually referred to as "nonlinear" or "vortex" lift.

Details of typical leading-edge vortex features are shown in figure 3. Lower surface flow forward of the dividing streamline separates at the leading edge, $S_1$, and forms the primary vortex. At moderate angles of attack, an upper-surface attachment line, $A_1$, is formed. Inboard of the upper-surface attachment line, air drawn over the leading-edge vortex attaches to the upper surface, forming a central potential flow region that is swept past the trailing edge. Outboard of $A_1$, flow drawn by the influence of the vortex is accelerated strongly toward the leading edge, until it passes beneath this primary vortex. The outward flow then enters a region of increasing pressure sufficient to cause a separation at $S_2$. As a result, a secondary vortex is formed and rotates opposite to the primary vortex. Flow over the secondary vortex reattaches at $A_2$ and continues on to the leading edge, where it separates again and joins the separated flow from the lower surface, to be swept into the primary vortex. Figure 3 also illustrates the low pressures induced by the primary and the secondary vortices.

This type of flow can exist on highly swept wings at supersonic as well as subsonic speeds, provided that the leading edge is swept greater than the angle weak shockwaves make with the freestream direction (i.e., subsonic leading edge).

In the analyses of leading-edge vortex flow, it has been found useful to correlate the data in terms of conditions normal to the leading edge. The incidence angle normal to the leading edge, $\alpha_N$, and the normal Mach number, $M_N$, are:

$$\alpha_N = \tan^{-1} \left( \frac{\tan \alpha}{\cos \Lambda} \right)$$

where $N = \text{freestream Mach number}$, $\alpha = \text{angle of attack}$, $\Lambda = \text{leading-edge sweep angle}$.

Experimental correlations have identified a boundary which indicates the conditions that determine whether attached or separated flow will occur on thin, highly-swept flat wings. This boundary is shown in figure 4. A round leading edge tends to suppress the formation of vortex flow and, thereby, shifts the separation boundary to higher incidence angles, as shown in figure 4.

The leading-edge vortex boundaries shown in figure 5 have been constructed using the empirical data of figure 4. This illustrates the Mach number limits where vortex flow can be expected to exist on highly swept wings with different sweep angles. This figure also shows that the thin, sharp flat wing separation boundary can be applied to wings with varying sweep angles by using the local leading-edge sweep angle.

The type of vortex flow described above is characterized by flow reattachment on the upper surface. All wings will eventually experience various types of breakdown that begin to limit the upper surface reattachment. Figure 6 (from reference 15) shows that the range of angle of attack, where reattached vortex flow occurs on highly swept delta wings, is limited for lower sweeps and higher aspect ratios by a phenomenon called "vortex bursting," whereas very highly swept delta wings (low aspect ratio) ultimately experience "vortex contact."

The bursting of a vortex refers to the change of flow pattern from a strong spiral motion about a small rapidly moving core to a weak slow rotational or turbulent motion about a large stagnant core. Upstream of bursting, axial velocities nearly 5 times freestream velocity have been measured near the primary vortex core. At bursting, the vortex core axial flow experiences a sudden deceleration to stagnation, and the core expands greatly about it. On extremely slender wings, the vortices from each leading edge can contact and eliminate flow reattachment, thereby reducing vortex lift.

The discussions in this paper are limited to conditions (Mach number, angle of attack) that permit vortex flow with reattachment on the upper surface.

The detailed characteristics of the flow field associated with the leading-edge vortices are indeed very complicated. Certain simplifying approximations must be made in order to construct mathematical models to be used to predict the effects of the leading-edge vortices.

The equations that are used to describe vortex flow on highly swept wings are the familiar small disturbance potential flow equations. The equations are linear. The solutions to the equations are, however, nonlinear and quite difficult to evaluate. The nonlinearity occurs because the solution must provide the strength and location of the
concentrated vortices, as well as the effects on the wing. Iteration techniques are required, starting from an initial guess for the strength and location of the vortices.

Some of the more successful prediction methods that have evolved are discussed in reference 29. These methods have been applied only to thin, highly swept wings where the vortex springs from the entire leading edge. An alternative approach, the suction analogy discussed in section 3 is of particular interest, since this forms the basis for the method presented in this paper.

3.0 Leading-Edge Suction Analogy

The leading-edge suction analogy formulated by E. C. Polhamus provides a simple technique to calculate vortex lift on flat sharp delta wing configurations.

This method has subsequently been extended to more arbitrary thin wing planform configurations through 28 and to the calculation of pitching moment and induced drag with vortex lift.

The suction analogy assumes that, if vortex flow with reattachment on the upper surface occurs, the total lift can be calculated as the sum of a potential-flow lift, \( C_{LP} \), and a vortex lift, \( C_{LV} \), associated with the leading-edge vortices. When vortex flow exists, the potential-flow lift term is due to the potential flow pressure acting normal to the wing surface. The potential lift for a flat wing is then given by the equation:

\[
C_{LP} = K_p \cos^2 \alpha \sin \alpha
\]

(1)

\( K_p \) is the lift curve slope given by small perturbation theory and depends only on the planform shape and Mach number. \( K_p \) can be calculated by any lifting surface theory.

The vortex lift is determined from an analogy between the force required to maintain flow about the spiral vortex and that required to maintain attached flow, as shown in figure 7.

For the attached flow condition, the flow ahead of the lower surface dividing streamline flows forward, and is accelerated around the leading edge to the top surface. The pressure required to balance the centrifugal force created by the flow about the leading edge results in the leading-edge suction force, \( C_S \), along the leading edge as

\[
C_S = 2 \int_0^1 c_s \, dn
\]

(5)

where \( n = y/(b/2) \) -- semispan fraction

\( C_S = \frac{2 \pi}{\cos \alpha} \left[ C_c(n) \left| \frac{1}{\tan \Lambda} C_1^2(n) \right| \right] \)

\( C_c(n) = \text{local chord} \)

\( S_{Ref} = \text{reference wing area} \)

\( K = \sqrt{1 - \frac{M^2 - 1}{\tan^2 \Lambda}} \)

\( C_1(n) \) is the local strength of the leading-edge pressure singularity and is obtained as the limit:

\[
C_1 = \lim_{\Delta C_p \to 0} \frac{\Delta C_p}{\frac{4}{	an \Lambda} \sqrt{X - X_{LE}}} \quad \text{as} \quad X \to X_{LE}
\]

(7)

\( \Delta C_p = \text{net lifting pressures} \)

\( X - X_{LE} = \text{local streamwise distance from the wing leading edge} \)

For a tapered wing planform, equation 6 becomes

\[
c_s = \frac{2 \pi}{\cos \alpha} \left[ \frac{1 - (1 - \Lambda \Lambda n)}{1 + \Lambda} \right] K \frac{\tan \Lambda}{\tan \Lambda} C_1^2(n)
\]

(8)

The local leading-edge suction can be calculated by the technique shown in figure 8.

Equation 3 and equation 5, together with equations 6 or 8, provide equivalent forms for calculating the leading-edge suction and, hence, the vortex lift on sharp thin wings. The leading-edge singularity strength, \( C_1 \), varies with \( \sin \alpha \). Hence, the effect of angle of attack is to change the magnitude, but not the
shape, of the vortex lift distribution in the streamwise direction.

The drag due to lift for thin sharp-edge flat wings is calculated from the potential and vortex lift as

\[ C_{Df} = (C_{Lp} + C_{Lv}) \tan \alpha \]  

\[ \alpha = \text{angle of attack} \]  

The pitching moment is calculated assuming that the streamwise distribution of potential lift is not affected by the vortex flow, and that the vortex lift acts normal to the wing near the leading edge.

The suction analogy has been shown to provide accurate estimates of lift, induced drag, and pitching moment for a wide range of sharp-edge flat wing configurations through 28, should be noted that there are limitations in providing wing surface pressure distributions.

The suction analogy, previously applied only to thin sharp-edge highly swept wings, has been extended to account for the effects of wing airfoil shape and thickness on the progressive growth of leading-edge vortices on flat wings. This method, described in detail in reference 29, is summarized in the next section.

4.0 Prediction of Airfoil Shape Effects

Experimental studies have shown that wing thickness has a retarding effect on the growth of leading-edge vortices. The experimental results indicate that, because of thickness, the vortex forms at an angle of attack greater than zero deg. The vortex then grows with reduced strength relative to a very thin wing at the same angle of attack. The retarding effect depends not only on the thickness distribution, but also on whether the airfoil nose is pointed or round.

4.1 Pointed Nose Airfoils

Experimental results that were obtained on a simple delta wing with various symmetric and asymmetric wedge shape thickness distributions indicate that vortex flow is largely independent of the shape of the lower surface.

Conversely, these results imply that the vortex formation, on a wing with a sharp-edge airfoil, is dependent only on the shape of the upper surface. With this key assumption, the effects of pointed nose airfoil thickness can be estimated by the following simple technique:

1. The flow is assumed to remain attached until the upper surface of the wing is at an angle of attack, \( \alpha \), as though the vortex were starting to grow on a thin wing at an effective angle of attack, \( \alpha_E \), equal to the actual angle of attack minus the upper surface nose angle.

\[ \alpha_E = \alpha - \delta_N \]  

\[ \alpha_E = 0 \quad \text{for} \quad | \alpha | < | \delta_N | \]  

The nose angle used in equation 10 for a symmetric wedge configuration is equal to half the included angle. An area-averaged nose angle is used for curved pointed airfoils.

2. The vortex lift for a flat constant section wing is then calculated as:

\[ C_{Lv} = K_v \cos \alpha \sin^2 \alpha_E \]  

3. The total lift is the sum of the potential lift (equation 1) and the vortex lift (equation 11).

4. The total lift is the sum of the potential flow pitching moment plus pitching moment associated with the reduced vortex strength. The calculation of drag due to lift is discussed in section 7.0.

Figure 9 contains a comparison of predicted lift and pitching moment, using this approach, with test data. Additional test versus theory comparisons, shown in reference 29, appear to substantiate this simple approach.

This technique can be applied to pointed nose wings with varying thickness by calculating the local suction coefficient, \( C_s \), at a local effective angle of attack \( \alpha_E(n) \):

\[ \alpha_E(n) = \alpha - \delta_N(n) \]  

\[ n = \text{semispan station} \]

In this case, the vortex does not spring uniformly from the leading edge, but moves progressively inboard as the angle of attack exceeds the local upper surface slope.

4.2 Round Nose Airfoils

Round nose airfoils on highly swept wings reduce the adverse pressure near the leading edge of highly swept wings. This effect is greatest over the inboard portion of the wing. Leading-edge separation in this case starts near the wing tip and moves progressively inboard with increasing angle of attack.

The straightforward method that was devised to account for the retarding effect of the round nose airfoil is summarized in figure 10, and includes the following steps:

a. Sections of the wing perpendicular to the leading edge are approximated locally by parabolas. The local parabolic nose drag, \( C_{Dp} \), is then calculated following the technique described in references 32, 33 and 34. The parabolic nose drag for a tapered wing is:
where $R_{l/c}$ is the ratio of the local perpendicular nose radius to the local streamwise chord.

b. The local parabolic nose drag is then compared with the local leading-edge suction coefficient given by equation 8.

c. As previously mentioned, the local leading-edge singularity strength, $C_1$, varies with "$\sin \alpha$".

$$C_1(\alpha) = C_{1,\text{REF}} \left( \frac{57.3 \sin \alpha}{C_{\text{REF}}} \right)$$

(14)

The vortex is assumed to begin to grow locally when the suction force, $c_s$, exceeds the parabolic nose drag, $C_{p_h}$. Combining equations 13, 14, and 8, this local separation angle, $\alpha_s$, can be expressed as:

$$\alpha_s = \pm \sin^{-1} \left( \frac{C_{1,\text{REF}}}{57.3 \frac{C_{p_h}}{C_{\text{REF}}}} \right)^{1/2} \frac{R_{l/c}}{C_{\tan \Lambda}}$$

(15)

where $C_{1,\text{REF}}$ is the local leading singularity strength calculated by linear theory at the reference angle of attack, $C_{\text{REF}}$.

d. The vortex is assumed to grow locally when the angle of attack exceeds the local separation angle, $\alpha_s$. The local vortex strength for a tapered wing is given by the effective suction coefficient, $C_{\text{SEFF}}$, as:

$$C_{\text{SEFF}} = \frac{2n}{\cos \Lambda} \left[ \frac{1 + (\lambda - 1)\eta}{1 + \lambda} \right] \tan \Lambda C_1^2(\alpha - \alpha_s)$$

(16)

$C_{\text{SEFF}} = 0$ for $|\alpha| < |\alpha_s|$.

$C_{\text{SEFF}}$ is the local sectional vortex lift.

"Sign($C_1$)" accounts for positive or negative lift depending if the vortex fails on the upper or lower surface of the wing.

Typical spanwise variations of the parabolic nose drag and the leading-edge suction are shown in figure 10 for a delta wing to illustrate this calculation concept. The vortex is predicted to exist on areas of the wing where the suction force exceeds the parabolic nose drag. On the remainder of the wing, the flow is attached.

The total lift is calculated as the sum of the potential flow lift plus the vortex lift obtained by integration of the local vortex lift along the wing leading edge. The pitching moment is calculated in a similar manner, again assuming that the streamwise potential flow lift distribution is unchanged by the vortex.

Figure 11 is a sample comparison from reference 29 of predicted lift and pitching moments using this method with test data for a 60-deg delta wing with a rounded nose airfoil section. The results indicated that the theoretical method for calculating the effects of round nose airfoil on vortex lift does properly predict the progressive development of the leading-edge vortex, as well as the vortex lift and pitching moment on highly swept flat wings. Furthermore, this method also indicates areas on a highly swept wing where attached flow theory applies.

5.0 Extension to Warped Wings

The elegance of the Polhausen suction analogy method is that linear theory is used to successfully predict the nonlinear forces associated with leading-edge vortices.

The local strength of the leading-edge pressure singularity, $C_1$, which is used to determine the leading-edge suction, $c_s$, is calculated from a linear theory pressure distribution. The nonlinearity in the suction analogy occurs because the leading-edge suction depends on "$\sin \alpha$" and also because exact "$\sin \alpha$" and "$\cos \alpha$" relations are used. An additional nonlinearity was introduced by the methods in the previous section in accounting for the retarding effects of pointed nose and round nose airfoils on the progressive development of the leading-edge vortex.

Since the essential element in this calculation, $C_1$, is linear, the methods for predicting the round nose or pointed nose airfoils effects on vortex development can readily be extended to arbitrary highly swept warped wings that are cambered, twisted, or have deflected leading-edge or trailing-edge devices.

The lifting pressure distribution on an arbitrary cambered and twisted wing at angle of attack and with flap deflections can be calculated as the linear sum of the following elements:

- A flat wing at angle of attack
- The cambered and twisted wing at zero angle of attack
- A flat wing with flap deflections at zero angle of attack

Similarly, the leading-edge pressure singularity can be calculated as the scaleable sum of the aforementioned elements as:

$$C_1(\eta) = C_{1,\text{REF}} \left( \frac{57.3 \sin \alpha}{C_{\text{REF}}} \right) + C_{1,\Gamma}(\eta) \left[ \text{TFAC} \right]$$

(18)

$$C_{1,\Gamma}(\eta) = \left( \frac{57.3 \sin \delta_{\Gamma}}{C_{\text{REF}}} \right)$$

where $\eta$ = semispan station.

$C_{1,\Gamma}(\eta)$ = the local leading-edge singularity strength for the flat wing at the reference angle of attack, $C_{\text{REF}}$. 

...
The local leading-edge singularity strength for the cambered and twisted wing at zero angle of attack

\[ C_{1T}(\alpha) = \text{the local leading-edge singularity strength for the cambered and twisted wing at zero angle of attack} \]

\[ C_{1\delta}(\alpha) = \text{the local leading-edge pressure singularity strength for the flat wing at zero angle of attack with "i th" surface deflected on angle } \delta \]

"sin α" scales the flat wing contribution to other angles of attack. Similarly, "TFAC" and "sin δN" are used to scale the camber/twist, and flap deflection contributions, respectively.

For simplicity, let the sum of the warped surface contributions to the leading-edge singularity strength be denoted "ΔC_1".

\[ \Delta C_1(n) = \text{TFAC } C_{1T}(\alpha) + C_{1\delta}(\alpha) \frac{57.3 \sin \delta_N}{\delta_{N_{REF}}} \]  

(19)

For warped wings with round nose airfoils, the leading-edge vortex is still assumed to occur when the local leading-edge suction, \( c_s \), equals the local nose pressure force, \( c_p \).

Combining equations 8 and 13, the vortex develops locally when

\[ C_1(\alpha) = \frac{\sqrt{R/L}}{k \tan A} \]  

(20)

Combining equations 18, 19, and 20, we obtain two possible vortex separation angles:

\[ \alpha_{SU}(\alpha) = \text{the angle of attack at which the vortex appears on the upper surface of the wing} \]

\[ \alpha_{SL}(\alpha) = \text{the angle of attack at which the vortex appears on the lower surface of the wing} \]

\[ \alpha_{SU}(\alpha) = \sin^{-1} \left[ \frac{\alpha_{REF} \frac{\sqrt{R/L}}{k \tan A} - \Delta C_1}{57.3 C_{IF}(\alpha)} \right] \]  

(21)

\[ \alpha_{SL}(\alpha) = \sin^{-1} \left[ \frac{\alpha_{REF} \frac{\sqrt{R/L}}{k \tan A} - \Delta C_1}{57.3 C_{IF}(\alpha)} \right] \]  

(22)

where \( \alpha_{SU}(\alpha) \) and \( \alpha_{SL}(\alpha) \) are the angles at which the vortex appears on the upper and lower surfaces of the wing, respectively.

The angle of attack range for which the round airfoil locally retards the growth of the leading-edge vortex is "\( \alpha_{SU}(\alpha) - \alpha_{SL}(\alpha) \)."

The effect of wing warp is to shift the local angle of attack range for which the leading-edge vortex is retarded by an amount of "\( \alpha_0 \)." This shift is obtained by setting the leading-edge radius to zero in either equation 21 or 22

\[ \alpha_0(\alpha) = \sin^{-1} \left[ \frac{\Delta C_1(\alpha)}{C_{IF}(\alpha)} \frac{\alpha_{REF}}{57.3} \right] \]  

(23)

The local leading-edge suction for selected values of TFAC and "N" (i.e., fixed warped wing shape) is then calculated using equation 6 with an effective angle of attack, \( \alpha_{EFF}(\alpha) \), given as:

- no vortex ---- 
  \( \alpha_{EFF}(\alpha) = 0 \) if \( \alpha_{SL}(\alpha) \leq \alpha_{SU}(\alpha) \)
- vortex on upper surface ---- 
  \( \alpha_{EFF}(\alpha) = \alpha - \alpha_{SU}(\alpha) \) if \( \alpha > \alpha_{SU}(\alpha) \)
- vortex on lower surface ---- 
  \( \alpha_{EFF}(\alpha) = \alpha - \alpha_{SL}(\alpha) \) if \( \alpha < \alpha_{SL}(\alpha) \)

The local vortex lift, \( C_{LV} \), is then calculated from equation 17.

This previously described calculation procedure can be easily used to conduct rapid parametric analyses on warped highly swept wings with round nose airfoils. The technique is shown in Figure 12.

First, potential flow analyses are made for the specified configuration at the desired Mach number. Calculations include:

- Flat wing analyses at \( \alpha_{REF} \)
- Cambered and twisted wing analyses at zero deg angle of attack
- Flat wing with the flaps deflected, \( \delta_N \)

The calculated results include the potential flow lift curve slope \( (k_p) \), the potential flow aerodynamic center \( (\frac{\partial C_L}{\partial \alpha} \alpha_{MP}/\partial C_L) \), and the lifting pressure distributions for each of the above configurations.

The leading-edge singularity strength, \( C_1 \), is computed for each of the aforementioned lifting pressure distributions.

For selected flap deflections and camber/twist scaling factors, the total warped wing singularity strength is calculated from equation 19.

The leading-edge singularity distributions, potential flow lift curve slope, and aerodynamic center are then used to calculate the vortex formation angle along the wing \( (\alpha_{SU}, \alpha_{SL}) \) using equations 20 and 21. The vortex lift, distribution, total lift, and pitching moment are then calculated.

The final three steps can be calculated on a programmable calculator. Thus, parametric analyses can easily be made to provide an understanding of wing geometry effects on leading-edge vortices.

Detailed analyses of pressure and force data, obtained on two NASA arrow-wing models, were then made to evaluate this procedure for predicting the progressive development of leading-edge vortices on warped wings with round nose airfoils. Results of these investigations are described in the next section.

6.0 NASA Arrow-Wing Models

A wind tunnel test of an arrow-wing body configuration consisting of flat and twisted
wings, as well as a variety of leading- and trailing-edge control surf ce deflections, was conducted at Mach numbers from 0.4 to 1.1 to provide an experimental data base for comparison with theoretical methods. The wings had the same planform and airfoil shapes. The pressure and force data obtained on the models provided an extensive set of pressure, force, and flow visualization data to test the calculation procedure for predicting the development and growth of leading-edge vortices on warped wings with round nose airfoils.

The model geometry is shown in figure 13. This figure also contains test data showing the variation of net lifting pressure, \( \Delta C_{L} \), with angle of attack at a number of spanwise stations near the leading edge for both the flat and the twisted wings. \( \Delta C_{L} \) equals the lower surface pressure minus the upper surface pressure at the same location on the wing.

The pressures grow linearly with angle of attack and then suddenly "break" or depart from this linear variation. The break angle varies across the span. The predicted local vortex development angles agree well with the experimental break angles for both wings.

The results of this study are summarized in figure 14. This figure compares the characteristics of the test pressures, together with the theoretical vortex development boundaries. The pressures vary linearly with angle of attack in the predicted attached flow region. The pressure "breaks" occur as the angle of attack exceeds the predicted local vortex development angles.

Note that the method predicts the angle of attack range for which the round nose airfoil retards the formation of the leading-edge vortex, as well as the shift in this attached flow region to higher angles of attack due to wing twist.

Experimental isobars shown in figure 15 give evidence of the differences in the progressive development of a leading-edge vortex on the twisted wing relative to the flat wing. Note that the locus of the upper surface minimum pressure peaks appears to point directly to the predicted start of the leading-edge vortex.

A typical wing section and the wind tunnel model twist distribution are also shown in this figure.

Figures 16 and 17 contain comparisons of experimental pressure data and attached flow theoretical predictions from reference 35 for the NASA twisted arrow-wing model. The theoretical attached flow predictions employ a general method for the numerical solution of nonlinear three-dimensional boundary-value problems. The method exploits a panel technique to solve the small disturbance potential flow equation, satisfying boundary conditions on the actual configuration surface. Compressibility effects are approximated by the Gartrell rule. The method is incorporated into Boeing Computer program TEA-230. The method is referred to as "TEA-230" in the figures.

Figures 16 and 17 each contain distribution at two spanwise stations where attached flow is predicted to occur, and at two spanwise stations where vortex flow is predicted to occur. Attached flow theory is seen to agree well with the test data where attached flow is predicted to occur. The test pressures depart significantly from the attached flow theory calculation in areas where vortex flow is predicted to occur.

Comparisons between test section normal force data and predictions by two attached flow theories, TEA-230 and FLEXSTAB, are shown in figure 18. FLEXSTAB is the unified subsonic/supersonic panel technique that was developed by Boeing under NASA Ames sponsorship. This method is based on a constant pressure panel method for solving the linearized potential flow equations for supersonic and subsonic speeds with linear boundary conditions. Again, it is seen that attached flow theory predictions agree well with the test data on areas of the wing where vortex flow is not predicted to occur.

Figure 19 contains plots of lifting pressure variations with angle of attack measured at a number of spanwise stations for the flat wing, and for the flat wing with different leading-edge flap deflections.

The predicted vortex formation angles on the upper surface of the wing agree reasonably well with the "breaks" in the pressure curves. The lower surface pressure data with the leading-edge flaps deflected do not exhibit a sudden "break." Instead, the data show a slow progressive development of nonlinear flow. This is probably the result of flow separation in the flap crease on the lower surface.

Figure 20 compares predicted vortex separation boundaries for the flat wing upper surface for the range of tested Mach numbers (M = 0.4 to 1.1). Mach number is seen to have a negligible effect on the vortex formation. These predictions are in agreement with the test data.

Consequently, the remainder of the study reported herein is directed at low subsonic Mach numbers. The results are expected to be applicable to a range of Mach numbers, providing that the normal Mach number remains moderately subsonic.

The results shown in this section and the previous section indicate that the theoretical method for calculating the effects of round nose airfoil on vortex lift (section 4.0) does properly predict the progressive development of the leading-edge vortex on flat and warped wings. Furthermore, this method also indicates areas on a highly swept wing where attached flow theory applies.

The discussions so far have dealt primarily with the effects of wing geometry on vortex development, lift, and pitching moment. Airfoil effects on drag due to lift with vortex flow are discussed in the next section.

7.0 Drag Prediction

In 1974, E. C. Polhamus showed the author the
results of an interesting experimental analysis he had made. This analysis is shown in figure 24.

An experimental vortex normal force, $C_{LV}$, was defined as the difference between measured normal force, $C_{NEXP}$, and the calculated potential flow normal force, $C_{NP}$:

$$C_{LV} = C_{NEXP} - C_{NP}$$

(24)

where $C_{NP} = K_p \cos^2 \alpha \sin \alpha$

An experimental suction force, $C_{SEXP}$, was determined from the difference between the drag component of the experimental normal force, $C_L \tan \alpha$, and the experimental drag due to lift, $C_D - C_{D0}$ as:

$$C_{SEXP} = \left( C_L \tan \alpha - (C_D - C_{D0}) \right) \frac{1}{\cos \alpha \cos \alpha}$$

(25)

The algebraic sum of the experimental normal force, $C_{NV}$, plus the experimental suction, $C_{SEXP}$, was found to equal the theoretical potential flow suction force, $C_{Strag}$ for a flat wing, the theoretical suction force depends only on the planform shape and Mach number. The experimental data in figure 24 indicate, however, that, depending on the airfoil shape, part of the theoretical suction appears as a vortex lift force, and the remaining portion appears as a "residual" suction force that acts in the chord plane of the wing. Physically, this chord force results from the potential flow or vortex suction pressures acting on the forward nose section of the airfoil. Additional verifications of this idea have recently been shown by similar correlations in reference 43.

Assuming that the sum of the vortex lift plus the residual suction equals the potential flow suction, the methods presented in this paper can be used to calculate the drag due to lift with vortex flow on flat wings.

The vortex lift or vortex normal force can be calculated on flat wings with pointed nose or round nose airfoils by the methods described in section 4.0. The local potential flow leading-edge suction coefficient can be calculated from a potential flow analysis, as described in section 3.0. The "residual" suction can then be calculated as the difference between the full theoretical suction, $C_{SO}$, and the effective suction, $C_{Sff}$, which is converted into vortex lift. Hence, the drag due to lift, $C_{DL}$, can be calculated as:

$$\Delta C_{DL} = (C_{LP} + C_{LTV}) \tan \alpha - (C_{SO} - C_{SFF}) \cos \alpha \cos \alpha$$

(26)

Figure 22 illustrates the redistribution of the potential flow suction into residual thrust plus vortex lift as the origin of the vortex moves inboard on a 70-deg delta wing.

Theoretical predictions of the drag due to lift on 60-deg delta wings with a round nose airfoil and with a pointed nose airfoil were made using this equation. The drag predictions, along with the predicted drag component of the normal force, are compared with test data in figure 23. The predictions agree well with the test data, particularly for the wing with the round nose airfoil. The pointed nose airfoil data begin to depart from the theoretical predictions approximately where the lift data indicated vortex breakdown occurred on this wing (figure 9).

The drag due to lift data from figure 23 are shown in figure 24 as an achieved or recovered suction factor, $S_{REC}$. This better illustrates the effect of airfoil shape on drag due to lift.

$$S_{REC} = \frac{C_L \tan \alpha - AC_{GL}}{C_L \tan \alpha - CL/\sin \alpha}$$

(27)

The denominator represents the increase in the potential flow drag level due to full loss of leading-edge suction. The numerator represents the portion of the loss in leading-edge suction that is recovered by the combined effects of vortex lift plus residual suction.

This method calculates drag due to lift with vortex flow as the sum of a normal pressure force distributed over the wing surface plus a residual suction force distributed somewhere in the chord plane of the wing. The actual distributions are unknown, knowing the distributions is not necessary as long as the wing surface is flat.

Consequently, this drag prediction method has not been applied to warped wings or wings with deflected flaps. In these cases, it appears that the distributions of the forces must be known, since the surface slopes upon which the pressure forces act vary over the wing.

Consequently, an alternative approach was taken to assess the general efficiency of vortex lift production. The objective was to provide an assessment of the importance of controlling the vortex development on warped wings. This is discussed in the next section.

B.0 Minimum Drag Due to Lift Considerations

The calculated vortex lift production efficiency on a flat wing is shown in figure 25 as the ratio of net vortex lift to the net vortex drag change. The net vortex lift is equal to the vortex lift minus the lift component of the leading-edge suction. The net vortex drag includes a reduction in drag because of the lower angle of attack necessary to produce a given lift, plus an increase in drag due to the loss in attached flow leading-edge suction. The net vortex drag is seen to be nearly equal to the net vortex lift for a wide range of angle of attack and for various sweep angles. Hence, vortex lift production is seen to be rather inefficient.

Figure 26 compares the minimum drag due to lift factor for attached flow with the drag due to lift factor for vortex flow on a sharp thin wing. At high sweep angles, the attached flow drag due to lift increases very rapidly with leading-edge sweep. At very high angles of attack, this indicates that drag due to lift with vortex flow on thin sharp-edge wings can actually be lower than the minimum drag due to lift with attached flow. The test data in figure 27 show that drag due to lift values lower than attached flow minimum drag values have indeed been measured.
These results imply that if the sweep is less than approximately 80 deg, minimum drag due to lift occurs when the leading-edge suction force is oriented to act in the plane of the wing, \( \phi = 0 \). At very high sweep angles, lower drag due to lift is obtained when the suction force is converted in the vortex lift and normal to the plane of the wing, \( \phi = 90 \) deg.

A study was then made to determine the optimum orientation of the suction force or vortex lift force that would minimize drag due to lift for different sweep angles. In this study, it was assumed that the suction force remains constant and is merely rotated forward from a position normal to plane of the wing \( (\phi = 90 \) deg). At very high sweep angles, lower drag due to lift is obtained when the suction force is converted in the vortex lift and normal to the plane of the wing, \( \phi = 90 \) deg.

The results in figure 28 correspond to a sweep of 70 deg. The results indicate that rotating the vortex lift vector to act more in the plane of the wing will sacrifice lift but reduce drag. Minimum drag occurs when the vortex lift vector lies in the plane of the wing. The results also indicate that the benefits in reduced drag due to lift, associated with tilting the vortex lift vector forward, are greatest at the lower lift coefficients. This is consistent with the results of a detailed numerical optimization study reported in reference 22.

Similar results were obtained with an increased sweep angle of 76 deg as shown in figure 29.

The results obtained for a very high sweep angle \( \phi = 0.25 \), as shown in figure 30, differ significantly. The optimum vortex force orientation angle depends on the required lift coefficient. The optimum angle increases from zero deg at small lift to 90 deg at higher angles of attack.

The results of this study indicate:

- At sweep angles where attached flow minimum drag due to lift is less than the drag due to lift on a flat wing with vortex lift (see figure 26), rotating the vortex vector to act more in the plane of the wing will reduce drag
- At higher sweep angles, minimum drag due to lift at lower lift coefficients occurs when the vortex lift vector acts nearly in the plane of the wing. At higher lift coefficients, tilting the lift force up from the plane of the wing reduces drag
- Additionally, the results imply that sweep angles of interest for airplane applications (less than 76 deg) achieve minimum drag due to lift when the vortex formation is suppressed. This corresponds to the suction force acting in the plane of the wing \( \phi = 0 \).

The experimental data, and the theoretical vortex formation boundaries in figure 14 for a flat wing and for a twisted wing, show that it is quite difficult to retain fully attached flow out to the wing tip. A number of studies were therefore made to determine the relative effects of various wing geometry parameters on the formation of the leading-edge vortices, using the method presented in section 5.0. The objectives of these studies were twofold:

1. Determining which geometry parameters are most effective in retarding the vortex development
2. Determining the sensitivity of the drag due to lift to the amount of wing span on which a vortex occurs

The results of these studies are summarized in the remaining section of the paper.

9.0 Airfoil Nose Radius Study

The effects of nose radius were investigated by analyzing three different airfoil shapes with the same maximum thickness/chord ratio on a 70-deg delta wing. The study configurations are shown in figure 31. Two of the airfoil sections are NACA airfoil sections. One has a small nose radius and the other has a moderate nose radius. The third airfoil used in this study has a rather large nose radius corresponding to an advanced high-speed airfoil section. This is the section used on the NASA arrow wing experimental studies discussed in section 6.0. Results of this investigation are shown in figures 32, 33, 34, and 35.

Increased airfoil nose radius, as shown in figure 32, has a very powerful effect on retarding the development of the leading-edge vortex.

Predicted lift and drag curves are shown in figure 33. A large nose radius reduces lift, but simultaneously results in large reductions in drag due to lift.

The effect of nose radius on drag due to lift is better illustrated by the effective or net recovered suction, \( S_{REC} \), (defined by equation 27) in figure 34.

By combining the results shown in figures 34 and 35, we can determine the sensitivity of drag due to lift, \( S_{REC} \), to the origin of the leading-edge vortex, \( x_{0} \). The results are shown in figure 35. The drag sensitivity data in figure 34 collapse to a single curve independent of lift, angle of attack, or airfoil shape. The results indicate that the recovered suction, \( S_{REC} \), is uniquely dependent on the origin of the leading-edge vortex.

Furthermore, it is seen that as long as the vortex is restricted to the outer 50 percent of the wing, 50 percent of the maximum effective suction can be obtained. Hence, drag due to lift for this planform is seen to be rather
The effect of wing sweep on the attached flow characteristics on delta wings is shown in figure 36. The lift curve slope, \( K_p \), decreases rapidly with increasing wing sweep. Consequently, the drag due to lift increases with sweep.

The suction factor, \( K_p \), however, does not vary significantly with wing sweep. Vortex lift of thin sharp-edge wings is directly related to \( K_y \) (equations 2 and 3), hence, vortex lift on sharp-edge delta wings should be relatively insensitive to wing sweep. This is indeed the case, as shown by the data and theoretical estimates in figure 37. The analyses of the round nose delta wings with different sweep angles are shown in figures 38, 39, 40, and 41.

The results in figure 38 show that, at equal angles of attack, the leading-edge vortex moves slightly further inboard on the lower sweep wing. At equal lift coefficients, the vortex origin is at essentially equal spanwise stations for the different wing sweeps.

Lift, drag, and pitching moments for the different sweep wings are shown in figures 39 and 40. The results show that the lower sweep wing produces more lift and less drag due to lift. This is because of the higher aspect ratio and larger potential flow lift curve slopes. Both are increased as wing sweep is reduced.

Figure 40 again shows that airfoil thickness reduces vortex lift, and that the reduced vortex lift was equal for the wing sweeps.

Figure 41 shows the variations of the recovered suction, \( S_{REC} \), with angle of attack, sweep, and vortex origin station, were nearly the same for all the wing sweeps.

10.2 Wing Notch Ratio Study

Cutting out the aft area of a delta wing, as shown in figure 42, produces the familiar arrow-wing planform. Arrow-wing planforms can be identified by the notch ratio, \( \zeta \). The notch ratio, as shown in figure 42, is equal to the ratio of the cut out chord length, \( AC \), to the root chord of the parent delta wing, \( CR \).

\[ \zeta = \frac{AC}{CR} \]

The low-speed attached flow aerodynamic characteristics of notched wings are shown in figure 42. The study airfoil shape is also shown.

The results show that the lower sweep wing produces more lift and less drag due to lift. This is because of the higher aspect ratio and larger potential flow lift curve slopes. Both are increased as wing sweep is reduced.

The suction factors for the delta wing and two notched wings are compared in figure 43. The figures also show that the study airfoil retards the vortex development nearly equally on each of the study planforms.

The calculated lift, drag, and pitching moment curves for the study configurations are shown in figure 44.

The recovered suction ratios, \( S_{REC} \), are shown in figure 45. The notched wings all achieve the same amount of net recovered suction with the study airfoil at equal lift coefficients. The notched wings are even less sensitive to vortex flow on the outer portions of the wing than the delta wing.

10.3 Wing Taper Ratio Study

The effects of wing taper ratio were investigated with the same study airfoil as in the sweep and notch ratio studies. Figure 46 shows the geometry of the study configurations. The attached flow lift curve slopes, aerodynamic centers, leading-edge suction factors, and drag due to lift factors are also shown.

The tapered wings experience a vortex that develops on the streamwise tip. This vortex, which is in addition to the leading-edge vortex, is not affected by airfoil shape, since the chord is parallel to the stream direction. The tip vortex grows immediately as the angle of attack is increased from zero deg. The tip vortex also contributes to the vortex lift. The tip vortex lift, \( C_{LVSE} \), can be calculated in terms of a side-edge suction factor, \( K_{VSE} \), as:

\[ C_{LVSE} = K_{VSE} \sin^2 \alpha \cos \alpha \]

The leading-edge suction factor, \( K_y \), decreases rapidly with taper ratio. The analyses in reference 25 indicate that, as the leading-edge suction factor \( K_y \) decreases, the side-edge suction factor, \( K_{VSE} \), increases such that the sum essentially remains constant with taper ratio. This again implies a consistency in leading-edge suction. The leading-edge suction lost on the leading edge appears on the side edge.

In this study, the side-edge suction factor was calculated as the difference between the delta wing suction factor and that calculated for the tapered wing. The side-edge vortex lift was then added to the net vortex lift calculated for the tapered wing, accounting for the round airfoil effects.

Figure 47 shows the nose pressure force distributions and suction distributions calculated for the three study wings. The nose pressure force is larger near the wing tip and smaller near the wing root, relative to the delta wing. This is due mainly to the difference in the corresponding chord lengths. The suction distribution is reduced in magnitude by wing taper. The peak of the suction distribution, however, moves outward as the taper ratio increases.

At equal angle of attack, the vortex moves further inboard on the delta wing than on the
tapered wings. At equal lift coefficients, the vortex development was equal for all three study wings.

The lift and drag calculations for the three wings are shown in figure 48. The lift is decreased and the drag is increased by taper ratio. The drag due to lift sensitivity calculations are shown in figure 49. Wing taper increases the amount of recovered suction. The vortex development rates, as indicated by the local separation angle distributions, differ significantly for each configuration. On the constant percent chord nose radius wing, the vortex is predicted to form at the tip at approximately 1.5 deg, and then move steadily inboard with increasing angle of attack. The cylindrical nose is predicted to retard the vortex growth up to approximately 3 deg. The vortex is then predicted to spring almost simultaneously from the outboard 50 percent of the wing.

The conical nose radius wing is predicted to be initially susceptible to vortex formation near the apex of the wing. It is not clear what happens, in reality, for this wing. The measured lift data for this wing closely match the zero thickness predictions. The recovered suction data for this wing are shown in figure 53. These data imply that the vortex is retarded on the conical nose radius initially for approximately 3 deg. The vortex then appears to spring almost simultaneously from the entire leading edge.

The results of this study seem to imply that increased nose radius is beneficial near the tip, but not at the expense of reduced inboard radius.

12.0 Wing Warp Effects

Warp investigations were made to determine the effect of wing warp on the formation of the leading-edge vortex on highly swept wings with round nose airfoils.

12.1 Wing Twist

Vortex separation boundaries were calculated for the arrow-wing planform shown in figure 54. The calculations were made for a flat and a twisted wing with zero thickness, and for a flat and a twisted wing with the study airfoil used in the previously described planform studies. The wing twist distribution resulted in a rather significant upward shift of the separation boundary. The maximum shift of 3 deg occurred near the wing tip. Wing wash-out is seen to be an effective way to retard the vortex growth, especially near the wing tip.

12.2 Flap Effects

Flap effects on vortex formation were calculated for an arrow-wing planform with the moderate nose airfoil. The flap considered in this study was a constant chord flap with the hinge line at 15 percent. A 15-deg flap deflection resulted in a 10-deg shift in the vortex boundary near the root, but only a 3-deg shift near the tip, as shown in figure 55.

Figure 56 contains results of a parametric study of the effect of leading-edge flaps on vortex formation boundary. Figure 57 shows the effectiveness of the simple leading-edge flaps on reducing the vortex flow areas on the study wing planform.

12.3 Wing Camber Effects

The qualitative effects of wing camber on the vortex development on thin, sharp, highly swept wings are shown in figure 58. At the design condition, camber aligns the leading edge with the local flow direction. The pressure gradients near the nose of the wing are very small, thereby eliminating leading-edge separation. At angle of attack above the design incidence, the lower surface dividing streamline moves aft on the wing. The pressure gradients increase sharply on the upper surface, leading to vortex formation.
This results in a displacement on the leading-edge separation boundary.

Lift, drag, and pitching moment data obtained on a flat delta wing and on a cambered delta wing are shown in figure 59. The calculations for the flat wing are also shown in this figure.

Camber is seen to have a powerful effect on reducing drag. This favorable effect, as shown by the recovered suction data in figure 60, indeed appears to come about through an upward shift in the separation boundary. So long as the normal Mach number is sufficiently subsonic, the beneficial effects of camber are obtained at subsonic and supersonic speeds. This is shown by the experimental data in figure 61.

Lift/drag comparisons for the flat and cambered wing are shown in figure 62. Camber is seen to significantly increase the lift drag ratio.

Wind tunnel data from the same test are presented in figure 63. These data imply that Reynolds number has an effect on the growth of the leading-edge vortex, but not on the initial formation angle.

13.0 Concluding Remarks

The foregoing discussions have shown that airfoil shape and thickness, nose radius, and wing warp have important effects on controlling the formation and strength of the leading-edge vortex on highly swept wings. Geometries that suppress the formation of the leading-edge vortex can result in considerable reductions in drag due to lift. The results of the planform studies imply that low drag due to lift can still be achieved if vortex flow is restricted to the outer portion of the wing.

A straightforward method was previously developed to predict the effect of airfoil geometry in the formation and spanwise growth of the leading-edge vortex. The method has now been extended to account for the effect of wing warp including twist, camber, and flap deflections.

The method can be used to predict the effects of airfoil shape, planform geometry, and wing warp on lift. The drag and pitching moment calculations are currently restricted to flat wings. The method also predicts portions on the wing where linear theory estimates are valid.

The method presented is simple and useful for both aerodynamic force estimates and parametric aerodynamic design studies.

The results of the drag due to lift minimization study indicated that, if vortex lift does occur, then tilting the lift vector forward towards the plane of the wing will reduce drag. This is true for wing sweeps of interest for airplane applications. On very highly swept wings, the optimum vortex orientation angle increases with increasing lift coefficient.

The majority of the studies presented in this paper were for low speeds. The methods and conclusions apply equally well to subsonic through supersonic speeds, provided the normal component of Mach number remains sufficiently subsonic.

More work is necessary to extend the method of predicting drag and pitching moment on cambered and twisted wings with flap deflections. Additional studies should be made to investigate effects of round nose airfoils on curved wing planforms, and wings with strakes or planform breaks. Additional systematic pressure, force, and flow visualization data are necessary to fully validate the method.

References

650 Delta Wings at Mach Numbers from 0.7 to 2.0," RAER & M No. 3305, July 1961


45. RIDDER, S-0, (1972) "Experimental Study of Induced Drag and Leading Edge Tangential Suction Force Spanwise Distribution of Thin Plane Delta Wings at Low Speeds Including The Effects of Fuselage Diameter," KTH AERO TN-58, 1972, Stockholm (2 References)
Figure 1. Types of Flow on Highly Swept Wings

- Wing sweep
- Airfoil shape
- Camber and twist
- Flap deflection
- Airfoil thickness
- Planform shape
- Dihedral
- Component integration
- Aeroelasticity

Figure 2. Factors Affecting Leading-Edge Vortex Development

- Mach number
- Reynolds number
- Angle of attack
- sideslip
- Maneuver

Figure 3. Typical Leading-Edge Vortex Features
SHARP LEADING-EDGE AIRFOILS

Figure 4. Leading-Edge Vortex Development on Thin, Highly Swept Flat Wings

FLAT WING LEADING-EDGE-SEPARATION BOUNDARIES

Figure 5. Effect of Wing Sweep on Flat Wing Leading-Edge Vortex Development

REF. POLHAMUS, E. C. (1971)

Figure 6. Limits of Reattached Vortex Flow—Sharp Edge Thin Delta Wing
LEADING-SUCTION APPROXIMATE ACp AS: ACp = c1 + c2 (i1) + c3 (i1/2) - c4 (i1/2)

LEAST-SQUARES FIT OF TO OBTAIN C1 + C2* + C3'*

FLAT WING
MACH = 0.85
α = 10 deg

Figure 7. Leading-Edge Suction Analogy—Thin, Sharp-Edge Wings

Figure 8. Leading-Edge Suction Calculation

TOTAL LIFT

Figure 9. Effect of Sharp Nose Airfoil Thickness on Vortex Lift of a 60-deg Delta Wing
(a) COMPUTE PARABOLIC NOSE DRAG

(b) COMPARE WITH LEADING-EDGE SUCTION

(c) VORTEX FORMS WHEN $C_S > C_R$

(d) VORTEX GROWS LOCALLY FROM $\alpha = \alpha_S$

Figure 10. Prediction of Round Nose Airfoil Effects on Leading-Edge Vortex Formation

Figure 11. Effect of Round Nose Airfoil on Vortex Lift of a 60-deg Delta Wing
Figure 12. Leading-Edge Vortex Parametric Analysis Procedure—Warped, Highly Swept Wing

Figure 13. Effect of Vortex Development on Leading-Edge Pressures
Figure 14. Comparison of Predicted Vortex Development With Experimental Leading-Edge Pressures

Figure 15. Comparison of Test Isobars and Predicted Vortex Lift
Figure 16. Comparison of Twisted Wing Attached Flow Theory Pressures With Test Data

Figure 17. Comparison of Twisted Wing Attached Flow Theory Pressures With Test Data (Concluded)

Figure 18. Effect of Vortex Development on Section Normal Force
Figure 19. Effect of Leading-Edge Flap Deflection on Vortex Development

Figure 20. Effect of Mach Number on Vortex Development—Upper Surface
Figure 21. Effect of Wing Profile on Leading-Edge Vortex Flow Characteristics

Figure 22. Effect of Vortex Origin on Potential Suction Redistribution
\[ \Delta C_{D_L} = (C_{L_P} + C_{L_V}) \tan \alpha - (C_{D_0} - C_{D_{EFF}}) \cos \alpha \cos \alpha \]

Potential Lift

Vortex Lift

Residual Suction

Figure 23. Drag Due to Lift Prediction Method

Figure 24. Drag Due to Lift Comparison of Sharp Versus Round Nose Airfoil

Figure 25. Vortex Lift Production Efficiency—Thin Flat Wing
Figure 26. Attached Flow and Vortex Flow Drag Due to Lift Comparison

Figure 27. Slender-Wing Drag Due to Lift

Figure 28. Effect of Vortex Lift Angle—70-deg Delta
Figure 29. Effect of Vortex Lift Angle—76-deg Delta

Figure 30. Effect of Vortex Lift Angle—AR = 0.25 Delta

Figure 31. Airfoil Nose Radius Study—Geometry
Figure 32. Airfoil Nose Radius Study—Leading-Edge Vortex Development

MACH = 0.2

Figure 33. Effect of Airfoil Nose Bluntness on Lift and Drag

Figure 34. Airfoil Nose Radius Study—Drag Due to Lift Sensitivity
Figure 35. Variation of Recovered Suction With Vortex Origin Station

Figure 36. Sweep Study—Geometry and Potential Flow Characteristics

Figure 37. Suction Analogy Predictions—Thin, Sharp-Edge Wings
Figure 38. Sweep Study—Leading-Edge Vortex Development

Figure 39. Sweep Study—Aerodynamic Characteristics

Figure 40. Sweep Study—Drag Due to Lift and Vortex Lift Comparisons
I

PLANFORM EFFECT

\[ S_{REC} \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ \alpha, \text{ DEGREES} \]

EFFECT OF LIFT AND ANGLE OF ATTACK

\[ S_{REC} \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ C_L \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

Figure 41. Sweep Study—Drag Due to Lift Sensitivity

PLANFORM GEOMETRY

NOTCH RATIO

LIFT FACTORS

K_D
K_F
K_L

NOTCH RATIO, \( \eta \)

ASPECT RATIO

AR
0.2
0.4

NOTCH RATIO, \( \eta \)

Figure 42. Planform Notch Ratio Study—Geometry and Potential Flow Characteristics

 SUCTION DISTRIBUTION

Figure 43. Planform Notch Ratio Study—Leading-Edge Vortex Development
Figure 4.1. Tetrahedron and Potential Flow Characteristics

- Aerodynamic Center
- Drag due to Lift
- Aspect Ratio
- Lift Factors
- Planform Geometry

- Lift Effect
- Angle of Attack Effect
- Planform Effect

Figure 4.2. Planform, Tetrahedron Study—Drag due to Lift Sensitivity

- Lift Ratio
- Notch Ratio
- Dendrites

Figure 4.3. Planform, Tetrahedron Study—Critical Parameters Characteristics

- Lift Ratio
- Notch Ratio
- Dendrites
Figure 47. Taper Ratio Study—Leading-Edge Vortex Development

Figure 48. Taper Ratio Study—Lift and Drag Comparisons

Figure 49. Taper Ratio Study—Drag Due to Lift Sensitivity
Figure 50. Delta Wing With Constant Percent Chord Nose Radius

Figure 51. Delta Wing With Cylindrical Nose Radius

Figure 52. Delta Wing With Conical Nose Radius Distribution
Figure 53. Effect of Nose Radius Distribution on Recovered Suction

Figure 54. Wing Twist Effects on Vortex Formation

Figure 55. Leading-Edge Flap Effects on Vortex Formation
Figure 56. Effect of Leading-Edge Flap Deflection on Vortex Development

Figure 57. Effect of Leading-Edge Flap Deflections on Vortex Flow Areas

Figure 58. Camber Effects on Leading-Edge Vortex Development on Sharp, Thin, Highly Swept Wings
Figure 59. Effect of Wing Camber on Lift, Drag, and Pitching Moment

M = 0.81
Re = 5.6 x 10^6

Figure 60. Effect of Camber on $S_{REC}$

M = 0.98
FLAT WING
CAMBERED WING

M = 0.81
FLAT WING
CAMBERED WING

Figure 61. Mach Number Effect on Cambered and Flat Wing Drag

DATA REF: NACA RM A55Q19

NACA 0003-63 AIRFOIL

NACA 0003-63
Figure 62. Effect of Camber on Lift/Drag Ratio

Figure 63. Effect of Reynolds Number on $S_{REC}$